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UNDERSTANDING THE ROLE OF NUCLEAR ELEMENTARY CHARGE AND ITS POTENTIAL IN ESTIMATING NUCLEAR BINDING ENERGY AND STRONG COUPLING CONSTANT

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Abstract

Considering our 4G model of final unification, there exists a nuclear elementary charge of magnitude *2.9464e* and strong coupling constant is the squared ratio of ordinary elementary charge to nuclear elementary charge. Nuclear elementary charge is having many applications in nuclear physics and other branches of physics like particle physics, super conductivity, condensed matter physics and unified physics. By refining the general nuclear binding energy formula, there is a possibility for defining a set of binding energy coefficients assumed to be linked with the strong coupling constant. By minimizing the error bars in the estimated binding energy, there is a chance for fixing the value of the strong coupling constant. Based on volume, surface, coulomb, asymmetry, pairing and congruent terms, it seems possible to fit the binding energy of isotopes of Z=1 to 137 and needs fine tuning for heavy isotopes of light proton numbers. Another interesting point is that, surface, coulomb and asymmetry energy coefficients can be considered as variable coefficients. Coulomb energy point of view, it is noticed that, increasing number of neutrons help in minimizing the nuclear radii against the coulomb repulsion. With reference to protons, neutrons and the coulomb energy, nuclear radii can be reviewed. With reference to an advanced binding energy formula and considering (2Z-1) and (3.5Z)+3 as the lower and upper mass limits, starting from Z=1 to 137, for the proposed 6 term semi empirical mass formula, RMS deviation is 3.25 MeV.

Keywords: 4G model of final unification; nuclear binding energy; nuclear radii; 6 term semi empirical mass formula; strong and electroweak mass formula; variable energy coefficients;

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1. Introduction

With reference to our 4G model of final unification, in our recent publications [1-12], we have developed a new formula for estimating nuclear binding energy [13-24] in terms of strong and electroweak interactions [25,26]. Our formula constitutes 4 simple terms and only one energy coefficient of magnitude 10.1 MeV. First term is a volume term, second term seems to be a representation of free nucleons associated with electroweak interaction, third term is a radial term and fourth one is an asymmetry term about the mean stable mass number. Considering this kind of approach, nuclear structure can be understood in terms of strong and weak interactions and complicated concepts like cold nuclear fusion can be understood in a theoretical approach positively.

In this short paper, we have presented a revised form of semi empirical mass formula (SEMF) with surface, coulomb and asymmetric energy coefficients as variables. This can be considered as a hybrid form of available SEMF. With a single set of energy coefficients, it seems to work for Z=1 to 137 and needs fine tuning for heavy isotopes of light proton numbers. In this context, we would like to emphasize the point that, strong coupling constant plays a vital role in fitting the nuclear binding energy coefficients. Clearly speaking, strong coupling constant can be inferred from nuclear binding energy coefficients. Considering our strong and electroweak mass formula and considering the proposed (revised) 6 term semi empirical mass formula, there is a chance to identify the strong coupling constant as a fundamental building block of atomic nuclei.

2. Three assumptions of 4G model of final Unification

Following our 4G model of final unification [1-12]

1) There exists a characteristic electroweak fermion of rest energy, $M_{wf}c^2 \cong 584.725$ GeV. It can be considered as the zygote of all elementary particles.
2) There exists a nuclear elementary charge in such a way that, $\left(\frac{e}{e_n}\right)^2 \cong \alpha_s \cong 0.1152$ = Strong coupling constant and $e_n \cong 2.9464e$.
3) Each atomic interaction is associated with a characteristic large gravitational coupling constant. Their fitted magnitudes are,

$G_e \cong$ Electromgnetic gravitational constant $\cong 2.374335\times10^{37}$ $m^3kg^{-1}sec^{-2}$

$G_n \cong$ Nuclear gravitational constant $\cong 3.329561\times10^{28}$ $m^3kg^{-1}sec^{-2}$

$G_w \cong$ Electroweak gravitational constant $\cong 2.909745\times10^{22}$ $m^3kg^{-1}sec^{-2}$

It may be noted that,

1) Weak interaction point of view [25,26], following our assumptions, Fermi's weak coupling constant can be fitted with the following relations.

$$\left.\begin{aligned} G_F &\cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong G_w M_{wf}^2 R_w^2 \cong 1.44021\times10^{-62}\ \text{J.m}^3 \\ \text{where,}\ &\begin{cases} R_0 \cong \dfrac{2G_n m_p}{c^2} \cong 1.24\times10^{-15}\ \text{m} \\ R_w \cong \dfrac{2G_w M_{wf}}{c^2} \cong 6.75\times10^{-19}\ \text{m} \end{cases} \end{aligned}\right\} \quad (1)$$

2) In a unified approach, most important point to be noted is that [10],

$$\hbar c \equiv G_w M_{wf}^2 \quad (2)$$

Clearly speaking, based on the electroweak interaction, the well believed quantum constant $\hbar c$ seems to have a deep inner meaning. Following this kind of relation, there is a possibility to understand the integral nature of quantum mechanics with a relation of the form, $n^2\hbar \equiv \dfrac{G_w\left(nM_{wf}\right)^2}{c}$ where $n = 1,2,3,..$ It needs further study with reference to EPR argument [27-30] and [1,31]. See section 5 for understanding the mystery of incompleteness of quantum mechanics in terms of hadronic mechanics [32-34]. String theory [35-39] can be made practical with reference to the three atomic gravitational constants associated with weak, strong and electromagnetic interaction gravitational constants. See Table 1. and Table 2. for sample string tensions and energies without any coupling constants.

Table.1 Charge dependent string tensions and string energies			
S.No	Interaction	String Tension	String energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94\times10^{10}$ N	$\sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_w}\right)} \cong 24.975$ GeV
2	Strong	$\frac{c^4}{4G_n} \cong 6.065\times10^{4}$ N	$\sqrt{\frac{e_n^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_n}\right)} \cong 68.79$ MeV
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505\times10^{-5}$ N	$\sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_e}\right)} \cong 874.3$ eV

Table.2 Quantum string tensions and string energies			
S.No	Interaction	String Tension	String energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94\times10^{10}$ N	$\sqrt{hc\left(\frac{c^4}{4G_w}\right)} \cong 292.36$ GeV
2	Strong	$\frac{c^4}{4G_n} \cong 6.065\times10^{4}$ N	$\sqrt{hc\left(\frac{c^4}{4G_n}\right)} \cong 273.3$ MeV
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505\times10^{-5}$ N	$\sqrt{hc\left(\frac{c^4}{4G_e}\right)} \cong 10234.77$ eV

2.1 Understanding the electroweak coefficient and nuclear stability

Our basic idea is that, all the nucleons are not participating in the nuclear binding energy scheme and non-participating nucleons can be called as 'Free nucleons'. These free nucleons revolve round the nuclear core. Each free nucleon reduces the nuclear binding energy by 10.1 MeV. Protons and neutrons jointly play a crucial role in fixing the number of free nucleons. Electroweak interaction is having a key role in understanding free nucleons and nuclear stability against beta decay. In this context, we noticed that,

$$\frac{m_p}{M_{wf}} \cong \left(\frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_z c^2)^0 (m_w c^2)^\pm}}\right) \cong \left(\frac{\sqrt{134.98\times139.57}\ \text{MeV}}{\sqrt{91187.6\times80379.0}\ \text{MeV}}\right) \cong 0.0016032 \qquad (3)$$

Here ratio of rest mass of proton to the assumed electroweak fermion is equal to the ratio of mean mass of pions to the mean mass of electroweak bosons. Based on this unique and concrete observation, we are very confident to say that, strong and weak interactions play a vital role exploring the secrets of nuclear structure. Based on this electroweak

coefficient 0.0016, stability corresponding to nuclear beta decay can be understood with the following relation.

$$\left.\begin{aligned} A_s &\cong 2Z + 0.0016(2Z)^2 \cong 2Z + 0.0064Z^2 \\ &\rightarrow \frac{A_S - 2Z}{4Z^2} \cong 0.0016 \end{aligned}\right\} \qquad (4A)$$

One can find a similar relation in the literature [16]. This relation can be well tested for Z=21 to 92. For example,

$$\frac{45-(2\times 21)}{4(21)^2} \cong 0.00170; \quad \frac{63-(2\times 29)}{4(29)^2} \cong 0.00149; \quad \frac{89-(2\times 39)}{4(39)^2} \cong 0.00181;$$

$$\frac{109-(2\times 47)}{4(47)^2} \cong 0.0017; \quad \frac{169-(2\times 69)}{4(69)^2} \cong 0.00163; \quad \frac{238-(2\times 92)}{4(92)^2} \cong 0.001595;$$

This is one best practical and quantitative application of our proposed electroweak fermion and bosons. Following this relation and based on various semi empirical mass formulae [13-21], by knowing any stable mass number, its corresponding proton number can be estimated with,

$$Z \cong \frac{A_S}{1+\sqrt{1+0.0064A_S}} \cong \frac{A_S}{2+0.0153A_S^{2/3}} \qquad (4B)$$

where $\frac{a_c}{2a_{asy}} \cong \frac{0.71\ \text{MeV}}{2\times 23.21\ \text{MeV}} \cong \frac{0.6615\ \text{MeV}}{2\times 21.6091\ \text{MeV}} \cong 0.0153$

Considering this relation, we are working on understanding stable super heavy elements.

2.2 Revised strong and electroweak mass formula

Based on the liquid drop model, we present our revised strong and electroweak mass formula [1]. It needs a review for accuracy and we are working in this new direction. For Z=6 to 125,

$$BE \cong \left(A - A_{free} - A_{radial} - A_{asym}\right)\left(B_0 \cong 10.1\ \text{MeV}\right)$$

$$\cong \left\{ \begin{array}{l} A - \left\{ \frac{1}{2} + \left[\Upsilon_Z \left[\left(Z + Z^{2/3} + Z^{1/3}\right)^2 + \left(N + N^{2/3} + N^{1/3}\right)^2 + \left(\frac{Z}{N}\left(Z^2\right)\right)\right]\right]\right\} \\ - A^{1/3} - \left(\beta \times \frac{\left(A_s - A\right)^2}{A_s} \right) \end{array} \right\} 10.1\ \text{MeV} \tag{5}$$

where, $A \times 10.1$ MeV represents the volume term

$A_{free} \times 10.1$ MeV represents the electroweak term

$A_{radial} \times 10.1$ MeV represents the radial term

$A_{asym} \times 10.1$ MeV represents the asymmetry term

$$\beta = 1 - \left(\frac{N - Z}{A} \right)^2$$

$$\Upsilon_Z \cong \Upsilon_0 \left[1 + \left(\Upsilon_0 Z\right)\right] \cong 0.000935\left[1 + 0.000935Z\right]$$

where $\Upsilon_0 \cong \sqrt{\frac{e}{e_n}}\left(0.001605\right) \cong 0.000935$

$$A_s \cong 2Z + 0.0064Z^2$$

$\cong$ Light house like (mean) stable mass number of Z.

$$B_0 \cong -\left(\frac{1}{\sqrt{\alpha_s}} \right) \frac{e^2}{8\pi\varepsilon_0\left(\mathrm{h}/m_p c\right)} \cong -\frac{e_n^2}{8\pi\varepsilon_0\left(G_n m_p / c^2\right)} \cong -10.1\ \text{MeV}$$

where $\begin{cases} \alpha_s = \text{Strong coupling constant} \cong 0.115 \text{ to } 0.12 \\ \mathrm{h}/m_p c = \text{Reduced Comption wavelength of proton} \\ G_n m_p / c^2 \cong 0.62 \times 10^{-15}\,\text{m} \end{cases}$

For evaluating the effectiveness of relation (5), we consider the following advanced relation as a reference [19].

$$BE \cong \left\{ \begin{array}{l} \left\{ \left[1 + \left(\frac{4k_v}{A^2} \right) |T_z| \left(|T_z| + 1 \right) \right] a_v * A \right\} + \left\{ \left[1 + \left(\frac{4k_s}{A^2} \right) |T_z| \left(|T_z| + 1 \right) \right] a_s * A^{\frac{2}{3}} \right\} \\ + \left\{ a_c * \left(\frac{Z^2}{A^{1/3}} \right) \right\} + \left\{ f_p * \frac{Z^2}{A} \right\} + E_p \end{array} \right\} \tag{6}$$

where, $T_z \cong$ 3rd component of isospin $= \frac{1}{2}(Z-N)$

$$\left.\begin{aligned} &a_v = -15.4963 \text{ MeV}, a_s = 17.7937 \text{ MeV} \\ &k_v = -1.8232, k_s = -2.2593 \\ &a_c = 0.7093 \text{ MeV}, f_p = -1.2739 \text{ MeV} \\ &d_n = 4.6919 \text{ MeV}, d_p = 4.7230 \text{ MeV} \\ &d_{np} \approx -6.4920 \text{ MeV} \end{aligned}\right\} \text{ and}$$

$$\left\{\begin{aligned} &\text{for } (Z, N) \text{ Odd}, E_p \cong \frac{d_n}{N^{1/3}} + \frac{d_p}{Z^{1/3}} + \frac{d_{np}}{A^{2/3}} \\ &\text{for } (\text{Odd } Z, \text{ Even } N), E_p \cong \frac{d_p}{Z^{1/3}} \\ &\text{for } (\text{Even } Z, \text{ Odd } N), E_p \cong \frac{d_n}{N^{1/3}} \\ &\text{for } (\text{Even } Z, \text{ Even } N), E_p \cong 0 \end{aligned}\right.$$

3. Revised semi empirical mass formula having 6 terms

With reference to relation (6) and considering surface, Coulomb and asymmetry energy coefficients as variables, we present our revised 6 term semi empirical mass formula.

Volume energy coefficient, $a_v \cong 16.0$ MeV

Surface energy coefficient, $a_s \cong \beta \times 19.4$ MeV where $\beta \cong \left[1 - \left(\frac{N-Z}{A}\right)^2\right]$

Coulombic energy coefficient,

$a_c \cong \left\{1 - \left(\frac{N-Z}{A}\right)^2\right\}^{-x} \times 0.71$ MeV where $x \cong 0.75 - \frac{Z}{2A}$

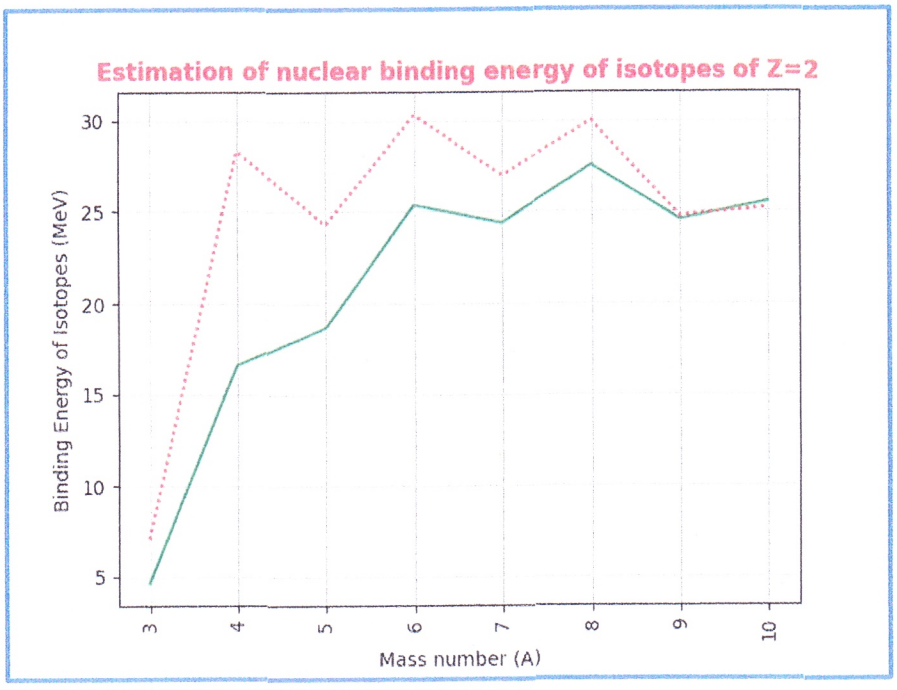
Asymmetry energy coefficient, $a_{asy} \cong \left[1 - \frac{1}{A}\right] \times 24.5$ MeV

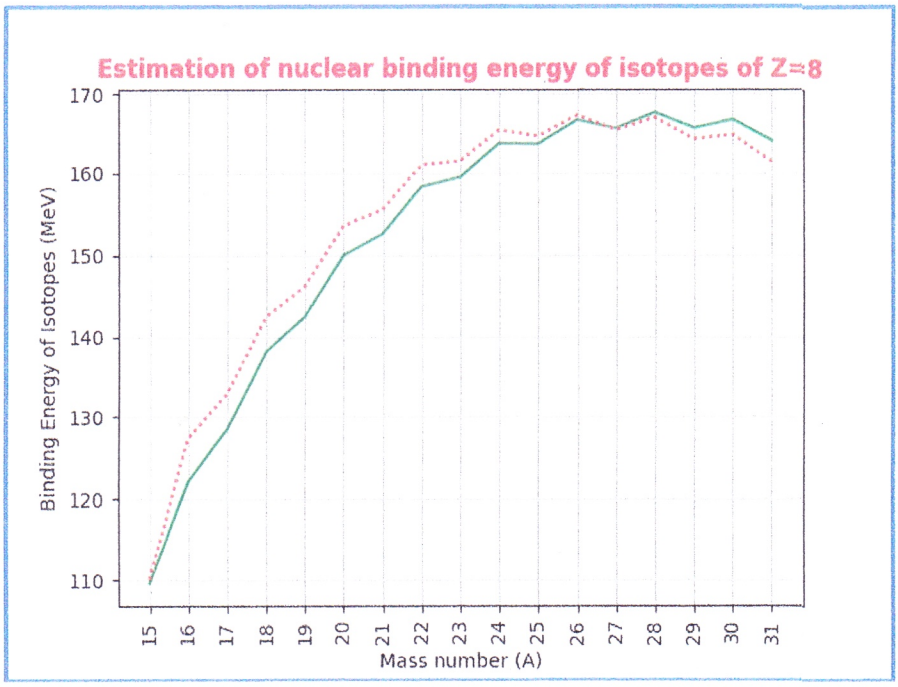
Pairing energy coefficient, $a_p \cong 10.0$ MeV

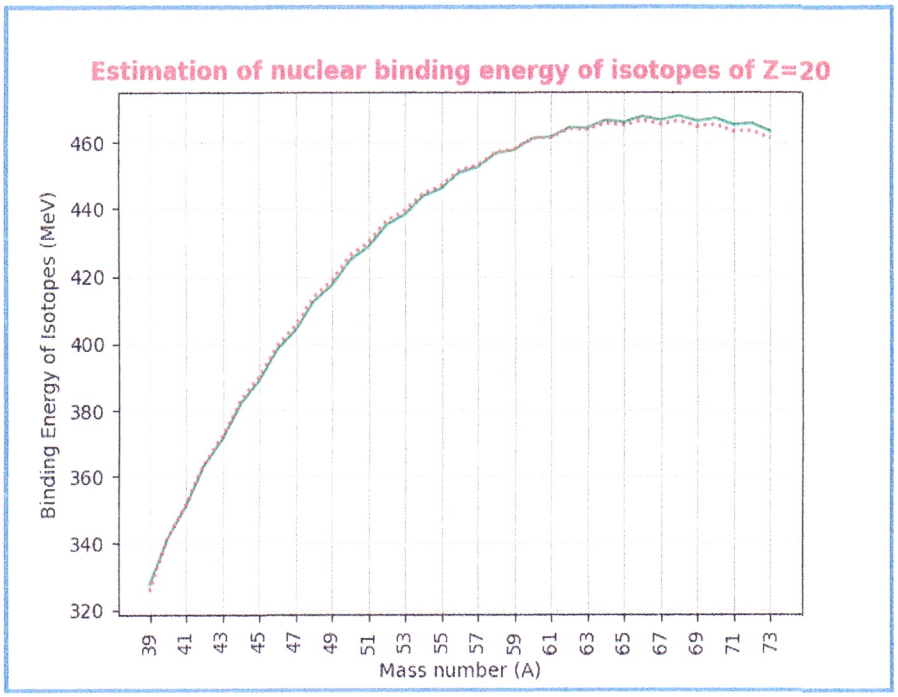
Congruent energy coefficient, $a_{cg} \cong 10.0$ MeV

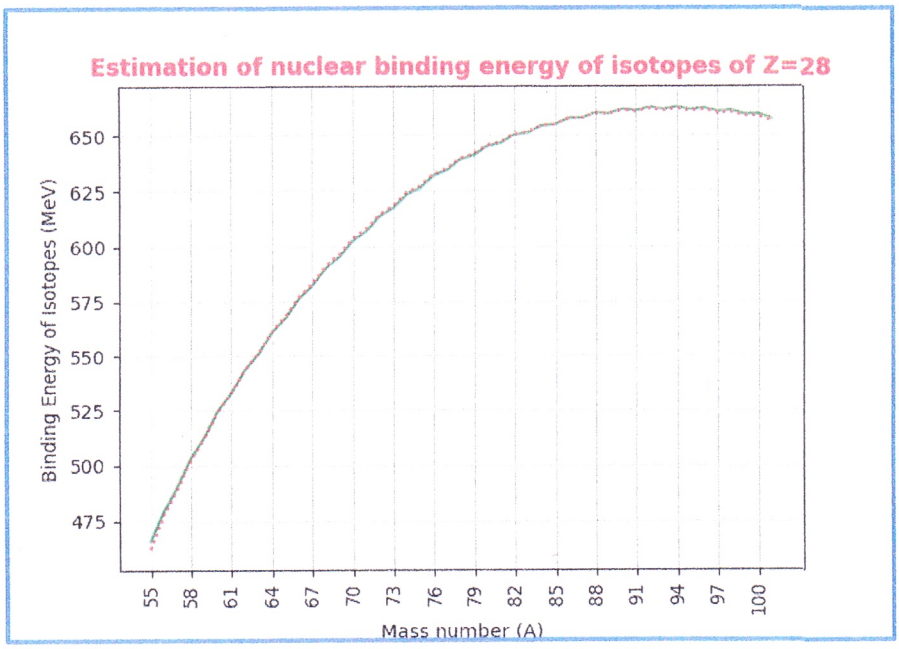
$$BE \cong [16.0\times A]-[\beta\times 19.4\times A^{2/3}]-\left[\frac{0.71\times Z^2}{\beta^x A^{1/3}}\right] \\ -\left\{\left[\left(1-\frac{1}{A}\right)24.5\right]\frac{(A-2Z)^2}{A}\right\}\pm\left[\frac{10}{\sqrt{A}}\right]+\left[10\times\exp\left(-4.2\frac{|N-Z|}{A}\right)\right]\Bigg\} \quad (7)$$

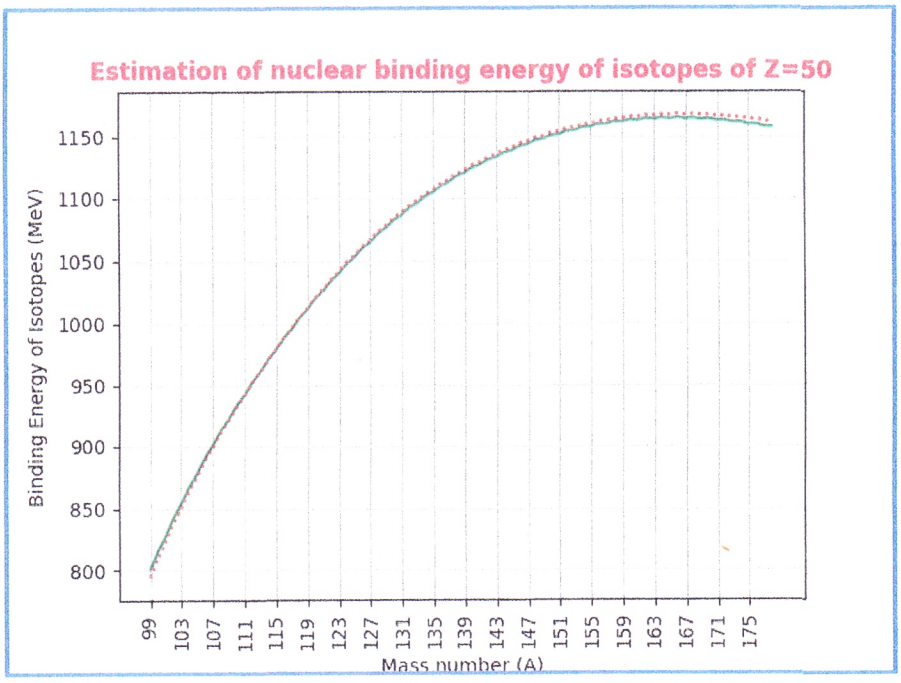
It seems to work from Z=1 to 137 without any difficulty and needs fine tuning for very heavy isotopes of very light proton numbers. See the following binding energy curves prepared for magic proton numbers [40], Z=2,8,20,28,50,82,100 and 114. Based on relation (6) and (7), red curve is our fit and green curve is the reference binding energy curve [19]. Interested readers can refer the nuclear binding energy data table as a supplementary material in our preprint, starting from Z=1 to 137 where lower and upper mass numbers are (2Z)-1 and (3.5Z)+3 respectively. (https://www.preprints.org/manuscript/202504.2431/v1). For our crude approximation, with reference to relation (6), for a wide range of protons and neutrons, relation (7) is having a root mean square deviation of 3.25 MeV.

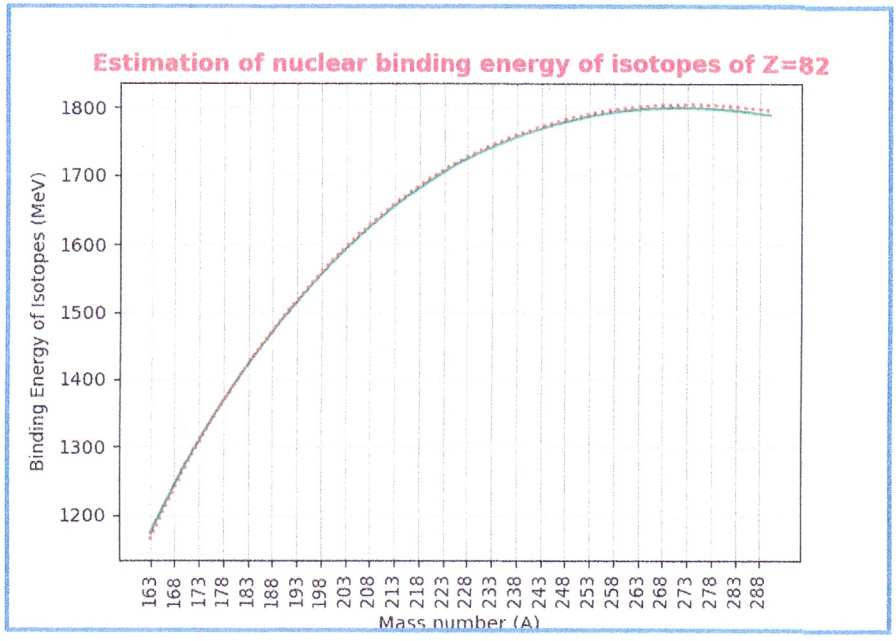


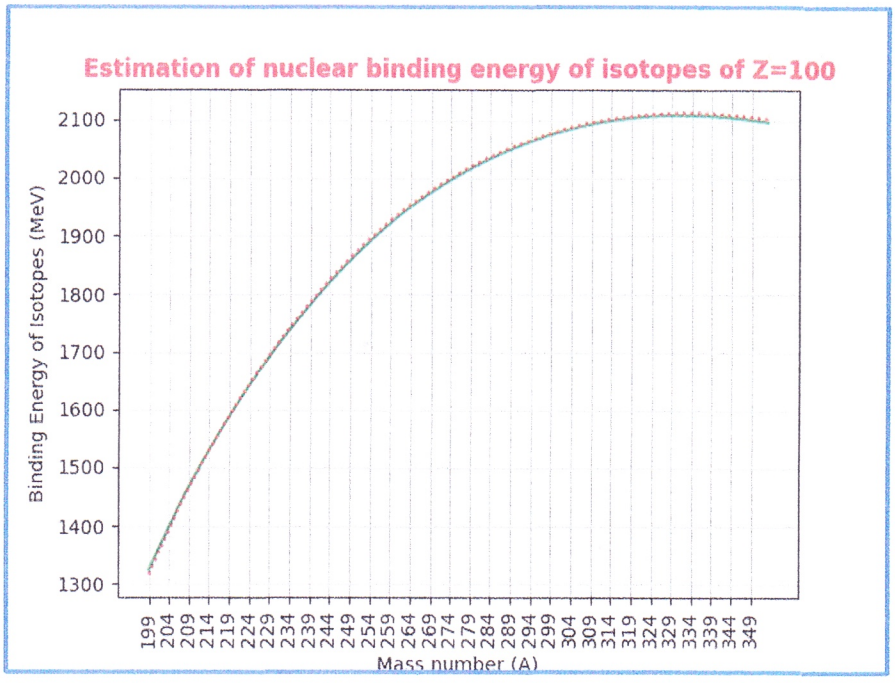

Estimation of nuclear binding energy of isotopes of Z=8
Binding Energy of Isotopes (MeV)
170
160
150
140
130
120
110
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
Mass number (A)

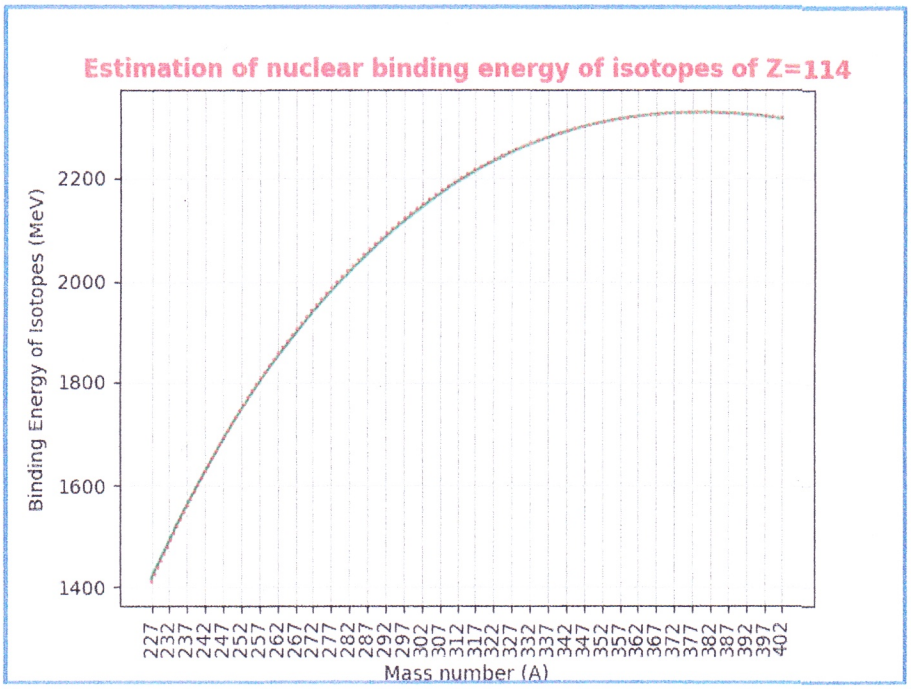

Estimation of nuclear binding energy of isotopes of Z=20
Binding Energy of Isotopes (MeV)
460
440
420
400
380
360
340
320
39
41
43
45
47
49
51
53
55
57
59
61
63
65
67
69
71
73
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=28
Binding Energy of Isotopes (MeV)
650
625
600
575
550
525
500
475
55
58
61
64
67
70
73
76
79
82
85
88
91
94
97
100
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=50
Binding Energy of Isotopes (MeV)
1150
1100
1050
1000
950
900
850
800
99
103
107
111
115
119
123
127
131
135
139
143
147
151
155
159
163
167
171
175
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=82
Binding Energy of Isotopes (MeV)
1800
1700
1600
1500
1400
1300
1200
163 168 173 178 183 188 193 198 203 208 213 218 223 228 233 238 243 248 253 258 263 268 273 278 283 288
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=100
Binding Energy of Isotopes (MeV)
2100
2000
1900
1800
1700
1600
1500
1400
1300
199 204 209 214 219 224 229 234 239 244 249 254 259 264 269 274 279 284 289 294 299 304 309 314 319 324 329 334 339 344 349
Mass number (A)



4. Short discussion on the energy coefficients of the revised semi empirical mass formula

Let $e_n \equiv$ Nuclear elementary charge $\cong 2.9464e$.

$R_{pp} \cong \dfrac{G_n m_p}{c^2} \cong 0.61965\,\text{fm} \cong$ Characteristic physical radius of proton

where, $\begin{cases} G_n \cong \text{Nuclear gravitational constant} \\ m_p \cong \text{Rest mass of proton} \end{cases}$

Nuclear potential associated with e_n and R_{pp} can be expressed as,

$$PE \cong -\frac{e_n^2}{4\pi\varepsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_{pp}} \cong -20.174\,\text{MeV} \qquad (8)$$

where, $\alpha_s \cong$ Strong coupling constant [41]

With reference to both nucleons and considering a characteristic potential energy as,

$$(PE)_{np} \equiv -\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong -40.35\ \text{MeV} \tag{9}$$

Based on this energy unit and beta decay, we consider the following relations for formulation of the energy coefficient. It needs further study.

1) Volume energy coefficient,

$$a_v \cong \left(\frac{m_e}{m_n - m_p}\right)\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\left(\frac{1}{2.531}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong 15.94\ \text{MeV}$$

2) Asymmetry energy coefficient,

$$a_{asy} \cong \left(1-\frac{1}{2.531}\right)\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\left(1-\frac{1}{2.531}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong 24.4\ \text{MeV}$$

3) Surface energy coefficient, $a_s \cong \dfrac{2a_v a_{asy}}{a_v + a_{asy}} \cong 19.3\ \text{MeV}$

4) Pairing energy coefficient, $a_p \cong \dfrac{e_n^2}{8\pi\varepsilon_0 R_{pp}} \cong \left(\dfrac{1}{\alpha_s}\right)\dfrac{e^2}{8\pi\varepsilon_0 R_{pp}} \cong 10.1\ \text{MeV}$

5) Congruence energy coefficient,

$$a_{cg} \cong \frac{e_n^2}{8\pi\varepsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{8\pi\varepsilon_0 R_{pp}} \cong 10.1\ \text{MeV}$$

6) Coulombic energy coefficient, $a_c \cong 0.7\ \text{MeV}$

Based on these energy coefficients, to a very good approximation, starting from Z=1 to 137,

$$\left.\begin{aligned} BE &\cong a_v A - \beta a_s A^{2/3} - a_c \frac{Z^2}{\beta^x A^{1/3}} - a_{asy}\frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} + a_{cg}\exp\left(-4.2\frac{|\mathrm{N}-\mathrm{Z}|}{\mathrm{A}}\right) \\ &\text{where}\quad \beta \cong 1-\left(\frac{\mathrm{N}-\mathrm{Z}}{\mathrm{A}}\right)^2 \text{ and} \\ x &\cong \left[0.75-\left(\frac{Z}{2A}\right)\right] \cong (0.5 \text{ to } 0.6) \rightarrow \text{Needs further study.} \end{aligned}\right\} \tag{10}$$

Points of interest are:

1) Characteristic nuclear radii linked with coulomb energy and nuclear binding energy [4,17] can be expressed as,

$$R_{(Z,N)} \cong \left[1-\left(\frac{N-Z}{A}\right)^2\right]^x A^{1/3} \times R_0$$

$$\text{where } \begin{cases} R_0 \cong (1.24 \pm \text{to } 0.01)\,\text{fermi} \\ x \cong 0.75 - \dfrac{Z}{2A} \approx (0.5 \text{ to } 0.6) \end{cases} \tag{11}$$

Considering $\left(A^{1/3}R_0\right)$, increasing neutron number, reduces the Coulomb repulsion between protons and thus helps in minimizing the radius by a factor $\left[1-\left(\frac{N-Z}{A}\right)^2\right]^x$ where $x \cong 0.75 - \frac{Z}{2A}$.

2) Considering the Fermi gas model of the nucleus [42,43],

a) Characteristic Fermi energy can be expressed as

$$E_F \cong \left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\frac{2e^2}{4\pi\varepsilon_0 R_{pp}} \cong \left(\frac{4}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \tag{12}$$

b) Volume and asymmetry energy coefficients can be expressed as,

$$\left.\begin{aligned} a_v \cong \left(1-\frac{3}{5}\right)E_F &\cong \left(1-\frac{3}{5}\right)\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{4}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_{pp}} \\ &\cong \left(\frac{8}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{\left(8\sqrt{3}\right)e^2}{4\pi\varepsilon_0 R_0} \quad \text{where}\left(\frac{1}{5\alpha_s}\right) \approx \sqrt{3} \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} a_{asy} &\cong \frac{3}{5}\left(\frac{2e_n^2}{4\pi\varepsilon_0 R_{pp}}\right) \cong \frac{3}{5}E_F \cong \left(\frac{6}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_{pp}} \\ &\cong \left(\frac{12}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{12\sqrt{3}e^2}{4\pi\varepsilon_0 R_0} \end{aligned}\right\} \tag{14}$$

$$\left(a_{asy}, a_v\right) \cong \left(\frac{10 \pm 2}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{(10 \pm 2)\sqrt{3}e^2}{4\pi\varepsilon_0 R_0}$$

c) Surface energy coefficient can be expressed as,

$$a_s \cong \frac{2a_v a_{asy}}{(a_v + a_{asy})} \cong \left(\frac{9.6}{5\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \frac{9.6\sqrt{3}e^2}{4\pi\varepsilon_0 R_0} \tag{15}$$

d) Congruent and Pairing energy coefficients can be expressed as

$$\begin{aligned} a_{cg} \cong a_p \cong \frac{E_F}{4} &\cong \left(\frac{e_n^2}{8\pi\varepsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{8\pi\varepsilon_0 R_{pp}} \\ &\cong \left(\frac{2}{\alpha_s}\right)\frac{e^2}{8\pi\varepsilon_0 R_0} \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \end{aligned} \tag{16}$$

See Table 3 for a rough estimation of our 6 term binding energy coefficients.

Table 3. Approximate energy coefficients of the revised 6 term semi empirical mass formula

a_v MeV	a_s MeV	a_c MeV	a_{asy} MeV	a_p MeV	a_{cg} MeV
15.94	19.3	0.70	24.4	10.1	10.1
16.0	19.4	0.71	24.5	10.0	10.0

Proceeding further, binding energy point of view, considering a radius value close to the the root mean square radius of proton or neutron [44,45], for light, medium and heavy atomic nuclides, above nuclear radii can be can be approximated with the following relation. It needs further study.

$$R_{(Z,N)} \cong \left[1 - \left(\frac{N-Z}{A}\right)^2\right]^x A^{1/3} \times 1.25\,\text{fm} \approx \left[\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right] \times 0.79\,\text{fm} \tag{17}$$

Clearly speaking, Coulomb energy term can be re-written as,

$$E_{cou} \cong -\frac{3Z^2 e^2}{20\pi\varepsilon_0 R_{(Z,N)}} \tag{18}$$

Binding energy expression can be re-written as,

$$BE \cong 16A - 19.4\beta A^{2/3} - \frac{1.09365 \times Z^2}{\left(\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right)} - \left[\left(1 - \frac{1}{A}\right) \times 24.5 \frac{(A-2Z)^2}{A}\right] \pm \frac{10}{\sqrt{A}} + 10 \times \exp\left(-4.2 \frac{|N-Z|}{A}\right)$$

where $\beta \cong 1 - \left(\frac{N-Z}{A}\right)^2$ and

$$\frac{3Z^2e^2}{20\pi\varepsilon_0 R_{(Z,N)}} \cong \frac{3Z^2e^2}{20\pi\varepsilon_0}\left[\left(\sqrt[3]{Z} + \sqrt[9]{Z^2N}\right) \times 0.79 \text{ fm}\right]^{-1} \quad \text{(19A)}$$

$$\left.\begin{aligned} BE &\cong 16A - 19.4\beta A^{2/3} - \frac{1.1 \times Z^2}{\left(\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right)} \\ &- \left[\left(1 - \frac{1}{A}\right) \times 24.5 \frac{(A-2Z)^2}{A}\right] \pm \frac{10}{\sqrt{A}} + 10 \times \exp\left(-4.2 \frac{|N-Z|}{A}\right) \\ &\text{where } \beta \cong 1 - \left(\frac{N-Z}{A}\right)^2 \end{aligned}\right\} \quad \text{(19B)}$$

Following relations (6) and (19), it can be understood that, radii of higher isotopes of higher proton numbers, seem to have a lower radii than the expected radii. See the following binding energy curves prepared for Z=28,50,82 and 114. Based on relation (6) and (19), red curve is our fit and green curve is the reference binding energy curve [19]. Based on the energy coefficients proposed in relation (7), and if $R_0 \cong 1.25$ fm, strong coupling constant [41] can be estimated as,

$$\alpha_s \cong \frac{8e^2}{20\pi\varepsilon_0 R_0 a_v} \cong 0.1151973 \text{ where } a_v \cong 16.0 \text{ MeV}$$

$$\alpha_s \cong \frac{12e^2}{20\pi\varepsilon_0 R_0 a_{asy}} \cong 0.1128463 \text{ where } a_{asy} \cong 24.5 \text{ MeV}$$

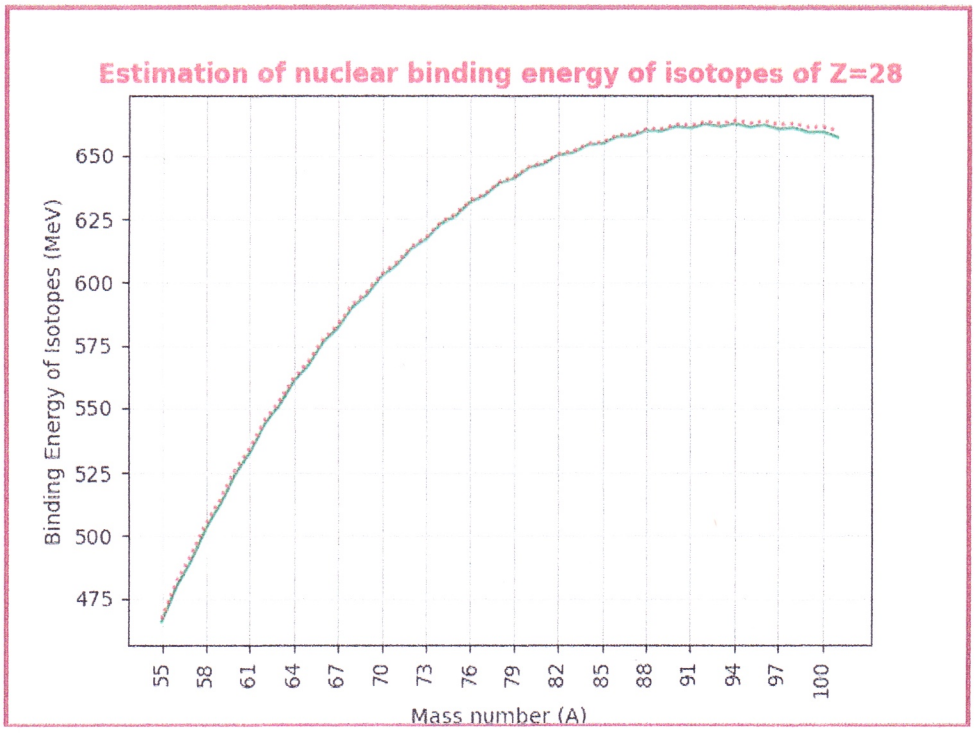
$$\alpha_s \cong \frac{9.6e^2}{20\pi\varepsilon_0 R_0 a_s} \cong 0.11401 \text{ where } a_s \cong 19.4 \text{ MeV}$$

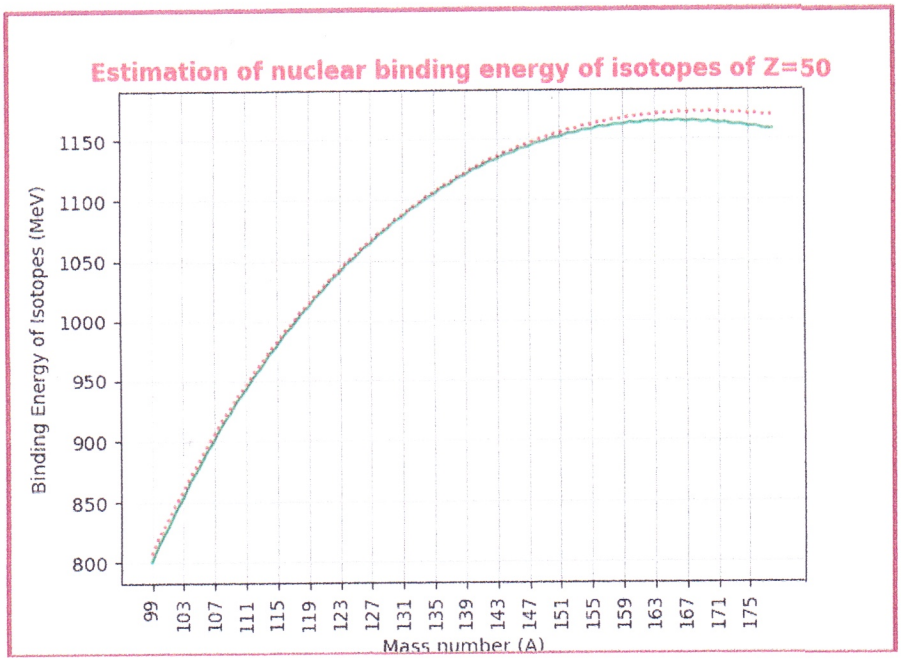
$$\alpha_s \cong \frac{e^2}{4\pi\varepsilon_0 R_0 a_{cg}} \cong \frac{e^2}{4\pi\varepsilon_0 R_0 a_p} \cong 0.1151973 \text{ where } a_{cg} \cong a_p \cong 10.0 \text{ MeV}$$

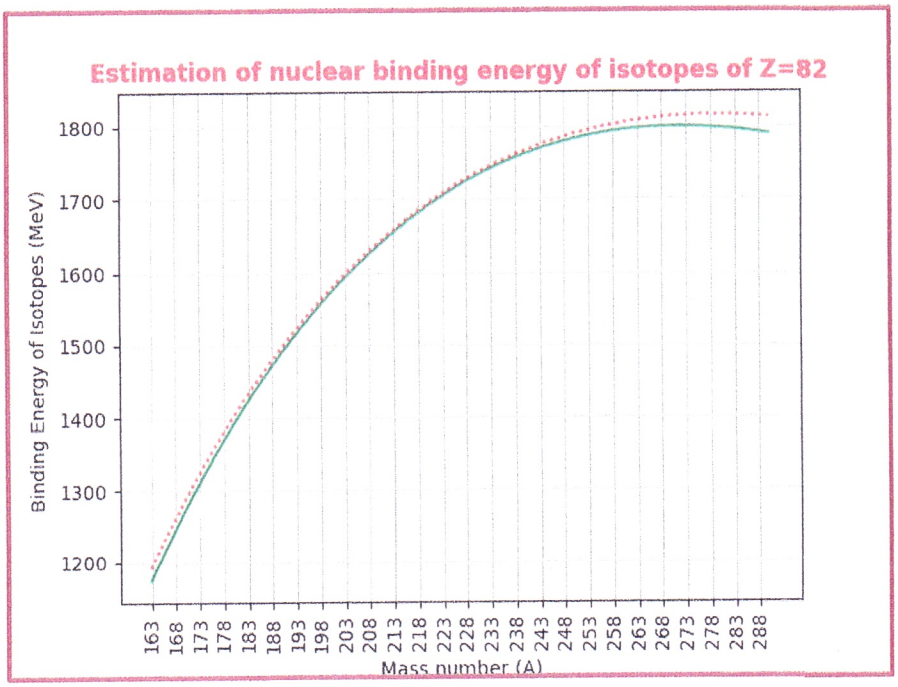
$$\alpha_s \cong \frac{1}{5\sqrt{3}} \cong 0.11547$$

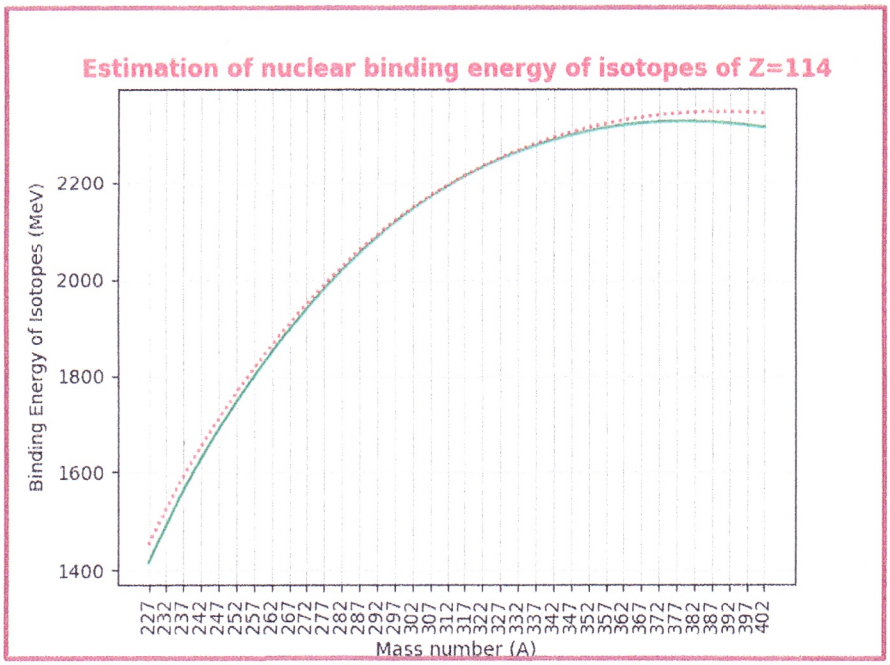
In a simplified and unified approach, approximately, it is possible to infer that,

$$\left.\begin{aligned} &(a_v, a_{asy}) \approx \left(\frac{2 \mp 0.4}{\alpha_s}\right)\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \\ &\frac{2a_v a_{asy}}{(a_v + a_{asy})} \geq a_s \leq \sqrt{a_v a_{asy}} \\ &a_{cg} \approx a_p \approx \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \\ &a_c \approx \left(\frac{3}{5}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \end{aligned}\right\} \qquad (20)$$




Estimation of nuclear binding energy of isotopes of Z=50
Binding Energy of Isotopes (MeV)
1150
1100
1050
1000
950
900
850
800
99
103
107
111
115
119
123
127
131
135
139
143
147
151
155
159
163
167
171
175
Mass number (A)


Estimation of nuclear binding energy of isotopes of Z=82
Binding Energy of Isotopes (MeV)
1800
1700
1600
1500
1400
1300
1200
163
168
173
178
183
188
193
198
203
208
213
218
223
228
233
238
243
248
253
258
263
268
273
278
283
288
Mass number (A)



5. Short discussion on R. M. Santilli's views on quantum mechanics and nuclear stability

By following the 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that quantum mechanics is not a complete theory [27-30], R. M. Santilli [32-34] identified a number of apparent insufficiencies of quantum mechanics in nuclear physics with particular reference to the lack of numerically exact representation (in one century of nuclear data), the prohibition by Heisenberg's uncertainty principle to represent the neutron synthesis from the electron and the proton in the core of stars despite their extremely big Coulomb attraction and the ensuing inability to represent the nuclear stability.

By following the below mentioned 4 points, he presented exact and time invariant representations of stable nuclei via the Lie-isotopic branch of hadronic mechanics and showed the necessity for the results of lifting Heisenberg's uncertainty principle for point-like particles into the isouncertainty principle of hadronic mechanics, also called Einstein's isodeterminism, for extended nucleons in condition of mutual penetration.

1) Considering protons and neutrons as isoparticles characterized by the isosymmetries and isorelativities of hadronic mechanics.
2) Representing all characteristics of the neutron at the nonrelativistic and relativistic levels as a hadronic bound state of an isoelectron and an isoproton.
3) Identifying the density of neutron in a way compatible with other experimental data.
4) Representing nuclear stability despite the natural instability of the neutron and despite the extremely repulsive protonic Coulomb forces.

It may be noted that, our approach of understanding quantum mechanics is based on 'weak gravity' and R. M. Santilli's approach is based on hadron mechanics. We are working on combining the two approaches for a better and simplified solutions.

6. Conclusion

Understanding nuclear binding energy with various physical terms and various coefficients is not a new point. With a single set of energy coefficients, our proposal helps in estimating the binding energy of all atomic nuclides starting from Z=1 to 137. Here, we would like to emphasize the point that, 4G model of the strong coupling constant (α_s) plays a vital role in understanding the origin of the binding energy coefficients and further study certainly helps in connecting high and low energy branches of nuclear physics and particle physics. Interesting point to be noted is that, all the binding energy coefficients, can be expressed as, $k\left[(2.95e)^2/4\pi\varepsilon_0(1.25\text{ fm})\right] \cong (k/\alpha_s)\left[e^2/4\pi\varepsilon_0(1.25\text{ fm})\right]$ where $k=(1.0\text{ to }2.5)$.

Data availability statement: The data that support the findings of this study are openly available.

Conflict of interest: Authors declare no conflict of interest in this paper or subject.

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